

# AS Maths Formulae Sheet

Shapes	
Area of Triangle	$\frac{1}{2} \times \text{base} \times \text{height}$
Area of Parallelogram	base $\times$ height
Area of Rectangle	length $\times$ width
Area of Trapezoid	$\frac{1}{2}(\text{sum of parallel sides}) \times \text{height}$
Circumference & Area: Circle	$c = 2\pi r, A = \pi r^2$
Cuboid Surface area	$SA = 2xy + 2xz + 2yz$ Where $x, y, \text{ and } z$ are side lengths
Cuboid Volume	$V = xyz$ where $x, y, \text{ and } z$ are side lengths
Cylinder Surface Area	$SA = 2\pi rh + 2\pi r^2$ Note: Curved part: $2\pi rh$
Cylinder Volume	$V = \pi r^2 h$
Cone Surface Area	$SA = \pi r l + \pi r^2$ Note: Curved part: $\pi r l$ where $l$ is slant length
Cone Volume	$V = \frac{1}{3}\pi r^2 h$
Sphere Surface Area	$SA = 4\pi r^2$ Note: Hemisphere: $2\pi r^2 + \pi r^2 = 3\pi r^2$
Sphere Volume	$v = \frac{4}{3}\pi r^3$ Note: Hemisphere: $\frac{2}{3}\pi r^3$
Prism Volume	$V = \text{Area of cross section} \times \text{height}$
Pyramid Volume	$V = \frac{1}{3} \times \text{base area} \times h$

Indices	
Multiplication	$x^a \times x^b = x^{a+b}$ $(x^a)^b = x^{ab}$ $(cx^a y^b)^d = c^d x^{ad} y^{bd}$
Division	$x^a \div x^b = x^{a-b}$
Negative Powers	$x^{-n} = \frac{1}{x^n}$
Fractions	$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$ $\left(\frac{x}{y}\right)^{-n} = \frac{y^n}{x^n}$
Rational Powers	$a^{\frac{n}{m}} = \left(a^{\frac{1}{m}}\right)^n = \left(\sqrt[m]{a}\right)^n$ $\frac{1}{a^{\frac{n}{m}}} = \sqrt[m]{a^{-n}}$

Series	
Binomial Theorem: integer powers	$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \dots + \binom{n}{n-1}a^1b^{n-1} + b^n$
Binomial Coefficient	$\binom{n}{r} = \frac{n!}{r!(n-r)!}$

Geometry	
Straight Line: Equation (gradient means slope)	<ul style="list-style-type: none"> <li>Slope intercept form: <math>y = mx + c</math></li> <li>General form: <math>ax + by + d = 0</math></li> <li>Point slope form: <math>y - y_1 = m(x - x_1)</math></li> </ul>
Parallel $\Rightarrow$ same slope	
Perpendicular $\Rightarrow$ "flip fraction and change the sign" (slopes multiply to make -1)	
Straight Line: Gradient	$m = \frac{y_2 - y_1}{x_2 - x_1}$
Distance between 2 points $(x_1, y_1), (x_2, y_2)$	$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Coordinates of midpoint of $(x_1, y_1), (x_2, y_2)$	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
Circles	$(x - a)^2 + (y - b)^2 = r^2$ centre $(a, b)$ , radius $r$

Quadratics	
Quadratic Function: Solutions to $ax^2 + bx + c = 0$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, a \neq 0$
Quadratic Function: Axis of Symmetry	$f(x) = x^2 + bx + c \Rightarrow x = -\frac{b}{2a}$
Quadratic Function: Discriminant	$\Delta = b^2 - 4ac$ <ul style="list-style-type: none"> <li><math>&gt; 0</math> (2 real distinct roots)</li> <li><math>= 0</math> (2 real repeated/double roots)</li> <li><math>&lt; 0</math> (no real roots)</li> </ul>
Completing The Square $ax^2 + bx + c = 0$	$a\left(x \pm \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$
Max/Min Value	$\frac{c - \frac{b^2}{4a}}{a}$
Exponential and Logarithmic Functions	$a^x = e^{x \ln a}$ $\log_a a^x = x = a^{\log_a x}$ where $a, x > 0, a \neq 1$
Exponentials & Logarithm Rules	<ul style="list-style-type: none"> <li><math>c \log_a b \Leftrightarrow \log_a b^c</math></li> <li><math>\log_a b = c \Leftrightarrow a^c = b, a, b, &gt; 0, a \neq 1</math></li> <li><math>\log_a b + \log_a c \Leftrightarrow \log_a bc</math></li> <li><math>\log_a b - \log_a c \Leftrightarrow \log_a \frac{b}{c}</math></li> <li><math>\log_a b \Leftrightarrow \frac{\log b}{\log a}</math></li> <li>Solving a power of <math>x</math>: log both sides if 2 terms or use substitution if 3 terms</li> <li>Solving an exponential: in both sides</li> <li>Solving a logarithm: raise e both sides or write as <math>\log_a</math>, as proceed as usual for log</li> </ul>

Transformations	
Transformations of $f(x)$	<ul style="list-style-type: none"> <li><math>a</math> = vertical stretch sf <math>a</math>,</li> <li><math>b</math> = horizontal stretch sf <math>\frac{1}{b}</math></li> <li><math>c</math> = translation <math>c</math> units <math>x</math> direction</li> <li><math>d</math> = translation <math>d</math> units in <math>y</math> direction</li> <li><math>f(-x)</math> = reflection in <math>y</math>, axis</li> <li><math>-f(x)</math> = reflection in <math>x</math> axis</li> </ul>

Trigonometry	
Sine Rule	Finding a side: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ Finding an angle: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
Cosine Rule	Finding a side: $a^2 = b^2 + c^2 - 2bc \cos A$ Finding an angle: $A = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right)$
Area of Triangle	$\frac{1}{2} ab \sin C$
Degrees $\leftrightarrow$ radians	D to R: $\times \frac{\pi}{180}$ R to D: $\times \frac{180}{\pi}$
Small Angle Approximations	$\sin \theta \approx \theta$ $\cos \theta \approx 1 - \frac{\theta^2}{2}$ $\tan \theta \approx \theta$
Pythagorean identity 1	$\sin^2 x + \cos^2 x = 1$
Cofunction	$\cos x = \sin(90 - x)$ $\sin x = \cos(90 - x)$
Identity of tan x	$\tan x = \frac{\sin x}{\cos x}$

Vectors: 2D vectors $\begin{pmatrix} a \\ b \end{pmatrix}$ year 1 and 3D vectors $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ year 2	
Vector Form	$a\mathbf{i} + b\mathbf{j} + c\mathbf{k} \equiv \begin{pmatrix} a \\ b \\ c \end{pmatrix}$
Properties (addition/subtraction, multiplication and scalar product)	$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \pm \begin{pmatrix} d \\ e \\ f \end{pmatrix} = \begin{pmatrix} a \pm d \\ b \pm e \\ c \pm f \end{pmatrix}$ $\lambda \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} \lambda a \\ \lambda b \\ \lambda c \end{pmatrix}$ $\begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} d \\ e \\ f \end{pmatrix} = ad + be + cf$ (last formula not in syllabus but useful to know)
Magnitude of a vector	$\left  \begin{pmatrix} a \\ b \end{pmatrix} \right  = \sqrt{a^2 + b^2 + c^2}$
Unit Vector	Unit vector of $\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \frac{1}{\sqrt{a^2 + b^2 + c^2}} \begin{pmatrix} a \\ b \\ c \end{pmatrix}$
Midpoint of $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ and $\begin{pmatrix} d \\ e \\ f \end{pmatrix}$	$\left(\frac{a+d}{2}, \frac{b+e}{2}, \frac{c+f}{2}\right)$
Scalar Product (not in syllabus but useful to know)	$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} d \\ e \\ f \end{pmatrix} = \left  \begin{pmatrix} a \\ b \\ c \end{pmatrix} \right  \left  \begin{pmatrix} d \\ e \\ f \end{pmatrix} \right  \cos \theta$ where $\theta$ is the angle between $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ and $\begin{pmatrix} d \\ e \\ f \end{pmatrix}$
Angle Between 2 vectors	$\theta = \cos^{-1}\left(\frac{\begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} d \\ e \\ f \end{pmatrix}}{\left  \begin{pmatrix} a \\ b \\ c \end{pmatrix} \right  \left  \begin{pmatrix} d \\ e \\ f \end{pmatrix} \right }\right)$
Vector Equation of a line (not in syllabus but useful to know)	$r = \begin{pmatrix} a \\ b \\ c \end{pmatrix} + \lambda \begin{pmatrix} d \\ e \\ f \end{pmatrix}$

Probability and Statistics	
Mean	If no frequency: $\bar{x} = \frac{\sum x}{n}$ , If frequency: $\bar{x} = \frac{\sum fx}{\sum f}$
Variance	If no frequency: $\sigma^2 = \frac{\sum x^2}{n} - \bar{x}^2 = \frac{\sum (x - \mu)^2}{n}$ If frequency: $\sigma^2 = \frac{\sum fx^2}{\sum f} - \bar{x}^2 = \frac{\sum f(x - \mu)^2}{\sum f}$ Note: can also use the formula $\frac{\sum x^2}{n} - \bar{x}^2$
Standard Deviation	$\sigma = \sqrt{\text{variance}}$
$s_{xx}$	$\sum (x_i - \bar{x})^2 = \sum x_i^2 - \frac{(\sum x_i)^2}{n}$
Probability of event A	$P(A) = \frac{n(A)}{n} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$
Complementary Events	$P(A) + P(\bar{A}) = 1$ i.e. probabilities add to 1
Combined Events (Addition Rule)	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Mutually Exclusive Events	$P(A \cap B) = 0$ Addition rule becomes: $P(A \cup B) = P(A) + P(B)$
Independent Events	$P(A \cap B) = P(A)P(B)$ Addition rule becomes: $P(A \cup B) = P(A) + P(B) - P(A)P(B)$ To find whether independent: Find $P(A), P(B)$ and $P(A \cap B)$ and see whether the former 2 multiply to make the latter or show that $P(A \cap B) = P(A)P(B)$
Binomial Distribution	$X \sim B(n, p)$ $E(X) = \text{Mean} = np, \text{Var}(X) = np(1 - p)$ $P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}$
Interquartile Range	$\text{IQR} = Q_3 - Q_1$
Outliers	Any values $> UQ + 1.5(\text{IQR})$ or $< LQ - 1.5(\text{IQR})$

Calculus (Differentiation and Integration)	
Turning/Stationary Points (Max/Min)	Solve $\frac{dy}{dx} = 0$
Proving whether Max/Min	$\frac{d^2y}{dx^2} > 0$ min and $\frac{d^2y}{dx^2} < 0$ max Or can do sign change test for $\frac{dy}{dx}$ using number line
Points of Inflection	Solve $\frac{d^2y}{dx^2} = 0$
Increasing/Decreasing	To find where increasing: solve $\frac{dy}{dx} > 0$ To find where decreasing: solve $\frac{dy}{dx} < 0$
Convex/Concave	To find where concave up/convex: solve $\frac{d^2y}{dx^2} > 0$ To find where concave down/concave: solve $\frac{d^2y}{dx^2} < 0$
Tangents and Normals	Differentiate to get $m$ (tangent means $\parallel$ , Normal means $\perp$ )
Implicit	"every time we differentiate $\frac{dy}{dx}$ we write $\frac{dy}{dx}$ "
Area between	curve & $x$ axis: $\int_a^b y \, dx$ curve & $y$ axis: $\int_a^b x \, dy$ (take + answer if neg) Between 2 curves: $\int_a^b (\text{top curve} - \text{bottom curve}) \, dx$ Remember to split up if separate areas
Differentiation 1 <sup>st</sup> Principles	$\frac{dy}{dx} = f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$
Chain Rule	$y = g(u), u = f(x) \Rightarrow \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
Product Rule	$y = uv \Rightarrow \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$
Quotient rule	$y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$
Derivatives	$x^n \Rightarrow nx^{n-1}$
Integrals	$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c, n \neq -1$

Mechanics	
SUVAT (5 formulae)	$v = u + at$ $s = \frac{(u+v)t}{2}$ $s = ut + \frac{1}{2}at^2$ $s = vt - \frac{1}{2}at^2$ $v^2 = u^2 + 2as$
Kinematics:	Distance: $\int_0^t v(t) \, dt$ , Displacement: $\int_{t_1}^{t_2} v(t) \, dt$ Velocity: $\int_{t_1}^{t_2} a(t) \, dt$ or $\frac{ds}{dt}$ Acceleration: $\frac{dv}{dt} = \frac{d^2s}{dt^2}$



